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Public Knowledge, Individual Human Capital, and Private Wealth in a Generalized Dynamic Walrasian General Equilibrium Theory

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Abstract: *This study is concerned with dynamics of a generalized Walrasian equilibrium theory with endogenous public knowledge, individual human capital, and private wealth. The model is a synthesis of a few well-established economic theories with Zhang's approach to modeling human behavior. It integrates the basic models in Walrasian general equilibrium theory, neoclassical growth theory, growth theory with endogenous human capital, and growth theory with endogenous knowledge. We build a model with* J *-group of households and three sectors. To describe motion of the economy is to solve* $2J+1$ *differential equations. We examine dynamic properties of the model of a 3-group economy with simulation. Comparative dynamic analysis is conducted to reveal transitory and long-term effects of some endogenous changes in policies, preferences, and efficiency of utilizing knowledge or human capital.*

Keywords: *growth, propensity to receive education; propensity to save; creativity; research policy; income and wealth distribution.*

JEL Classification: *O41, O42, O47.*

1. Introduction

There are many factors which may be important determinants for economic growth under different circumstances. In general, three factors can be reasonably considered as most important determinants of economic growth. They are human capital (qualified labor force), machines, and knowledge. Human capital is the basic factor for high labor productivity. Good machines enable to the economy to produce many things and supply different services which cannot be effectively conducted without tool. Knowledge is the basic determinant of modern knowledge-based economies. The importance of these three factors is well recognized. Modeling each of three factors results a main body of economic literature in dynamic economics. The purpose of this study is to build a general equilibrium model with heterogeneous households which treat the three factors as interactively endogenous variables. It integrates the basic models in Walrasian general equilibrium theory, neoclassical growth theory, growth theory with endogenous human capital, and growth theory with endogenous knowledge.

The model is based on neoclassical growth theory. The theory was a main concern of research in economic theory during the 1950s to the 1970s (Solow, 1956; Swan, 1956; Burmeister and Dobell, 1970; and Zhang, 2005). The core model for the development of the theory is Solow's one-sector growth model with homogeneous population. The main determinant of per capita income and consumption is capital accumulation. A key obvious limitation of the Solow model is the assumption of a fixed saving rate out of the disposable income. The Solow model was extended to multi-sector economies by many authors with an alternative Ramsey utility. But the theory has not been properly extended to economies with heterogeneous households. Neoclassical growth theory has failed to properly explain income and distribution issues with wealth accumulation. The Ramsey utility function upon which most of neoclassical growth models use is not an effective tool for modeling households' behavior. The popularity of the invalid approach to human behavior partly explains why modern economics provide light interesting insights into dynamic issues of income and wealth distributions. On the other hand, Walrasian general equilibrium theory deals with issues related to income and wealth distribution in a static world. Nevertheless, economists have not succeeded in generalizing the theory to include endogenous wealth accumulation. Zhang (2005, 2014) applies another approach to household behavior to integrate Walrasian general equilibrium theory and neoclassical growth theory. This study extends Zhang's model to include endogenous knowledge.

Another main stream of economics is to deal with dynamic interdependence between human capital accumulation and economic growth. The core model of this approach is Uzawa's two-sector growth model. The model is now generally referred as the Uzawa-Lucas two-sector growth model (Uzawa, 1965; Lucas, 1988). The model studies an economic with production and education sectors. The economy accumulates endogenously wealth and human capital. There are many further studies on education and growth on the basis of the Uzawa-Lucas model (e.g., Jones *et al.* 1993; Stokey and Rebelo, 1995; De Hek, 2005; Chakraborty and Gupta, 2009; and Sano and Tomoda, 2010). Households' preferences for education and for saving are important for sustainable economic growth. Uzawa's approach does take account of households' preferences for education. It is concerned with an economy with a single type of households which is described by the Ramsey approach. Line in the Uzawa model, we consider formal education as a sole channel of human capital accumulation. Education sector is in free markets. Applying his unique utility function approach to integrate the Uzawa-Lucas model, Zhang (2015) integrates the Uzawa-Lucas model with traditional neoclassical growth trade theory. This study is based on Zhang's model as far as modeling education and human capital accumulation are concerned.

Much of basic knowledge is public good, even though it takes time for newly innovated knowledge be publically freely available. There are many channels of knowledge accumulation. This study considers that knowledge production is solely conducted by research institutions which are financially supported by government. Modeling endogenous knowledge and economic growth has been conducted by economists over years (e.g., Romer, 1986; Grossman and Helpman, 1991; Aghion and Howitt, 1992, 1998). The endogenous growth theory is mostly concerned with knowledge creation, knowledge diffusion, and knowledge utilization without effectively analyzing the role of wealth accumulation and human capital accumulation. This study is interested not only in endogenous knowledge, but also in human capital and wealth accumulation. We examine interactions between wealth accumulation, knowledge and human capital in an economy with heterogeneous households. Knowledge growth affects education and economic production. Knowledge growth should be affected by people with human capital. Modeling knowledge with research sector in this study is based on Zhang's model (1993). We connect Zhang's model with endogenous knowledge with Zhang's model of human capital.

This study builds a growth model of heterogeneous households. The model is concerned with endogenous saving, human capital accumulation through education, and knowledge growth through research. We are interested in effects of households' differences in propensities to save, propensities to receive education, productivity of human capital accumulation, human capital application efficiency, creativity, and knowledge utilization efficiencies. The model is a synthesis of the Solow growth model, the Uzawa-Lucas two-sector growth model, Walrasian general equilibrium theory, Zhang's model with endogenous knowledge. As mentioned before, the model is based on Zhang's three models on integrating Walrasian general equilibrium theory (2014), on knowledge (1993), and on human capital (2015). The rest of the paper is organized as follows. In Section 2 we develop the model of heterogeneous households with endogenous wealth, human capital, and knowledge. In Section 3 we show properties of the model and illustrate the movement of the economy with three types of households. In Section 4 we conduct comparative dynamic analysis to show how the economy shifts its development paths when the system is subject to exogenous changes such as changes in propensities to save, creativity, and propensities to receive education. In Section 5 we conclude the study.

2. The global growth model with research and education

As in Funke and Strulik (2000), we examine interactions between knowledge, human capital, and wealth. Economic mechanism of wealth accumulation is based on neoclassical growth theory. Human capital due to education is based on the Uzawa-Lucas two-sector growth model. Knowledge growth is influenced by the endogenous growth models. Different from Funke and Strulik (2000), this paper applies Zhang's approach to household's behavior and knowledge growth. Zhang (1993) develops a growth model with endogenous capital and knowledge. The model introduces a research sector into the standard one-sector growth model. Knowledge is public good in Zhang's approach. Zhang (2015) proposes a growth model composed of heterogeneous household. The model includes endogenous growth mechanisms with endogenous wealth and human capital. This study synthesizes these two models to examine a dynamic interdependence between knowledge, human capital, wealth, and income and wealth distribution.

We build a national economy composed of heterogeneous households. There are J types of households, indexed by $j = 1, ..., J$. The economy has three sectors - one production/industrial sector, one education sector, and one research sector. The production sector is similar to the sector in neoclassical growth theory. There is only one durable goods in the economy. Saving is conducted by households. Assets are owned by households. Households distribute their disposable incomes between consumption and saving. All markets are perfectly competitive. All available capital and labor are fully utilized. All prices are measured in terms of the commodity. The price of commodity is unity. The production sector employs physical capital, labor, and knowledge to produce goods and services. Knowledge is public good and is accumulated through research.

Research sector is financially supported by the government through taxing households and production sector. Capital and labor are paid at their marginal rates. Education sector provides educational services. Human capital is accumulated through education. Education sector uses physical capital, labor, and knowledge as inputs. We introduce the following variables:

- i, e, and r - subscript index for production, education and research sectors, respectively;
- $\bar{N}_j$ - fixed population of group j;
- T_0 - available time for work and study;
- τ and $\bar{\tau}$ - fixed tax rate on production sector and $\bar{\tau} \equiv 1 - \tau$;
- τ_{jc} and $\tilde{\tau}_{jc}$ - fixed tax rate on group j's consumption and $\tilde{\tau}_{jc} \equiv 1 + \tau_{jc}$
- τ_{jw} and $\bar{\tau}_{jw}$ - fixed tax rate on wage income and $\bar{\tau}_{jw} \equiv 1 - \tau_{jw}$;
- τ_{jk} and $\bar{\tau}_{jk}$ - fixed tax rate on interest income and $\bar{\tau}_{jw} \equiv 1 - \tau_{jw}$;
- $r(t)$ and $w_j(t)$ - rate of interest and wage rate per unit time of one unit qualified labor service;
- $p(t)$ - price of education service per unit of time;
- $T_j(t)$ and $T_{je}(t)$ - work time and study time of the representative household of group j;
- $s_j(t)$ and $\bar{k}_j(t)$- saving made by and wealth owned by the representative household of group j;
- $c_j(t)$ - consumption level of goods by the representative household of group j;
- $Z(t)$ and $H_j(t)$ - knowledge stock and level of human capital of group j;
- $K(t)$ and $\bar{K}_j(t)$- capital stocks of the economy and wealth owned by group j; and
- $K_m(t)$ and $N_m(t)$- capital stock and qualified labor force employed by sector m, $m = i, e, r$.

The total labor supply

Each group's labor supply is the total qualified labor time of the population. The national labor supply is the sum of all group's labor supplies.

$$N(t) = \sum_{j=1}^{J} N_j(t) = \sum_{j=1}^{J} H_j^{m_j}(t) T_j(t) \bar{N}_j, \qquad (1)$$

where m_j is the representative household $j's$ efficiency of applying human capital.

Production function and marginal conditions of the production sector

The production function of the production sector is

$$F(t) = A_i Z^{m_i}(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ A_i, \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1, \tag{2}$$

in which A_i, α_i, and β_i are positive parameters. Here, the parameter m_i is called the production sector's knowledge utilization efficiency. Knowledge stock $Z(t)$ is public good in the sense that everyone is freely access to it and no one is excluded to fully use it when someone uses it. For any individual firm rate of interest, wage rate, and prices are exogenously given at each point in time. The production sector chooses $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions imply:

$$r(t) + \delta_k = \frac{\alpha_i \bar{\tau} F_i(t)}{K_i(t)}, \ \ w(t) = \frac{\beta_i \bar{\tau} F_i(t)}{N_i(t)}. \tag{3}$$

The current income and disposable income

The representative household's current income $y_j(t)$ from the interest payment $\bar{\tau}_{jk} r(t) \bar{k}_j(t)$ and the wage payment $\bar{\tau}_{jw} H^{m_j}(t) T_j(t) w(t)$ is

$$y_j(t) = \bar{\tau}_{jk} r(t) \bar{k}_j(t) + \bar{\tau}_{jw} H_j^{m_j}(t) T_j(t) w(t). \tag{4}$$

The representative household's disposable income $\hat{y}_j(t)$ is the sum of the current income and the value of wealth. That is

$$\hat{y}_j(t) = \bar{y}_j(t) + \bar{k}_j(t). \tag{5}$$

The disposable income is distributed between expenditures on saving, consuming, and receiving education.

The budget and utility function

This study applies Zhang's approach to household behavior to decide how the representative household rationally chooses how much to save $s_j(t)$, how many hours to receive education $T_{je}(t)$, and how much to consume $c_j(t)$. We have the following budget constraint

$$\tilde{\tau}_{jc} c_j(t) + s_j(t) + p_j(t) T_{je}(t) = \hat{y}_j(t). \tag{6}$$

The time constraint for each household is

$$T_j(t) + T_{je}(t) = T_0. \tag{7}$$

Insert (7) in the definition of $\hat{y}_j(t)$

$$\hat{y}_j(t) = \bar{y}_j(t) - \bar{\tau}_{jw} H^{m_j}(t) T_{je}(t) w(t), \tag{8}$$

where

$$\bar{y}_j(t) \equiv (1 + \bar{\tau}_{jk} r(t)) \bar{k}_j(t) + \bar{\tau}_{jw} H^{m_j}(t) T_0 w(t).$$

The variable $\bar{y}_j(t)$ is the "potential" disposable income that the household gets when the household spends all the available time on work. Substitute (8) into (6)

$$\tilde{\tau}_{jc} c_j(t) + s_j(t) + \bar{p}(t) T_{je}(t) = \bar{y}_j(t), \tag{9}$$

where

$$\bar{p}(t) \equiv p(t) + \bar{\tau}_{jw} H_j^{m_j}(t) w(t).$$

The left-hand side of (9) is the sum of the total cost of consumption, saving and opportunity cost of education. As in Zhang (2015), we specify the following household's utility function

$$U_j(t) = c_j^{\xi_{0j}}(t) s_j^{\lambda_{0j}}(t) T_e^{\eta_{0j}}(t), \tag{10}$$

where ξ_{0j} is called the propensity to consume, λ_{0j} the propensity to own wealth, and η_{0j} the propensity to receive education.

Optimal decision

The household maximizes $U_j(t)$ subject to (9). The first-order conditions imply

$$c_j(t) = \xi_j \bar{y}_j(t), \quad s_j(t) = \lambda_j \bar{y}_j(t), \quad \bar{p}_j(t) T_{je}(t) = \eta_j \bar{y}_j(t), \tag{11}$$

where

$$\xi_j \equiv \frac{\rho_j \xi_{j0}}{\tilde{\tau}_{jc}}, \quad \lambda_j \equiv \rho_j \lambda_{j0}, \quad \eta_j \equiv \rho_j \eta_{j0}, \quad \rho_j = \frac{1}{\xi_{j0} + \lambda_{j0} + \eta_{j0}}.$$

Wealth accumulation

The change in wealth is saving minus dissaving. From the definitions of $\bar{k}_j(t)$ and $s_j(t)$ we have

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{12}$$

The education sector

Education market is perfectly competitive. The student pays the education fee $p(t)$ per unit of time. The education sector uses capital input, labor input and knowledge to supply education service. The production functions of the education sectors are taken on the following form

$$F_e(t) = A_e Z^{m_e}(t) K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \quad m_e \geq 0, \quad \alpha_e, \beta_e > 0, \quad \alpha_e + \beta_e = 1, \tag{13}$$

where A_e, α_e and β_e are positive parameters. The parameter m_e is the efficiency of knowledge utilization by the education sector. The education sector pays teachers and capital with market rates. The total cost of the education sector is $w(t) N_e(t) + (r(t) + \delta_k) K_e(t)$. The marginal conditions imply

$$r(t) + \delta_k = \frac{\alpha_e \, p(t) F_e(t)}{K_e(t)}, \quad w(t) = \frac{\beta_e \, p(t) F_e(t)}{N_e(t)}. \tag{14}$$

Accumulation of human capital

We follow Uzawa (1965) in modelling human capital accumulation. We apply a generalized Uzawa's human capital accumulation as follows

$$\dot{H}_j(t) = \frac{\upsilon_{je} Z^{m_{jh}}(t) \left(H^{m_j}(t) T_{je}(t)\right)^{b_{je}}}{H^{\pi_{je}}(t)} - \delta_{jh} H_j(t), \tag{15}$$

where δ_{jh} (> 0) is the depreciation rate of group $j's$ human capital, υ_{je}, m_{jh}, a_{je}, and b_{je} are non-negative parameters. The sign of π_{je} may be negative or positive. The equation implies that human capital rises in education service per unit time and in the (qualified) total study time, $\left(H^{m_j}(t) T_{je}(t)\right)^{b_{je}}$. The term $1/H^{\pi_{je}}$ implies that learning through education may exhibit increasing returns to scale in the case of $\pi_{je} < 0$ or decreasing returns to scale in the case of $\pi_{je} > 0$.

Knowledge creation

Knowledge growth is through research. Knowledge stock rises in the knowledge stock, labor input and capital input. Following Zhang (1993), we specify knowledge changes as follows

$$\dot{Z}(t) = v_r Z^{m_r}(t) K_r^{\alpha_{0r}}(t) N_r^{\beta_{0r}}(t) - \delta_Z Z(t), \tag{16}$$

in which δ_Z (≥ 0) is the depreciation rate of knowledge, and v_r, m_r, α_{0r}, and β_{0r} are positive parameters.

Effective optimal research with the government budget

The governments collect taxes to support research sector. The government receives the following tax income $Y_p(t)$

$$Y_p(t) = \tau F_i(t) + \sum_{j=1}^{J} \left[\tau_{jc} c_j(t) \bar{N}_j + \tau_{jk} r(t) \bar{k}_j(t) \bar{N}_j + \tau_{jw} H_j^{m_j}(t) T_j(t) \bar{N}_j w(t) \right]. \tag{17}$$

The budget constraint for the research sector is

$$(r(t) + \delta_k) K_r(t) + w(t) N_r(t) = Y_p(t). \tag{18}$$

The total capital cost for the research sector is $(r(t) + \delta_k) K_r(t)$ and the total labor cost is $w(t) N_r(t)$.

The government spends the total budget on supporting research in such a way that the total research output $v_r Z^{m_r}(t) K_r^{\alpha_{0r}}(t) N_r^{\beta_{0r}}(t)$ be maximized. The research sector's problem is

$$Max\, v_r Z^{m_r}(t) K_r^{\alpha_{0r}}(t) N_r^{\beta_{0r}}(t)$$

subject to (18). The marginal conditions imply

$$(r(t) + \delta_k) K_r(t) = \alpha_r Y_p(t), \quad w(t) N_r(t) = \beta_r Y_p(t), \tag{19}$$

where

$$\alpha_r \equiv \frac{\alpha_{0r}}{\alpha_{0r} + \beta_{0r}}, \quad \beta_r \equiv \frac{\beta_{0r}}{\alpha_{0r} + \beta_{0r}}.$$

Demand and supply in national education market

The total demand for education service in group j is $T_{je}(t)N_0$. The demand and supply for education balances at any point in time

$$\sum_{j=1}^{J} T_{je}(\mathrm{t})\overline{N} = F_e(t). \qquad (20)$$

Full employment of national labor and capital

The labor force is distributed between the two sectors. The national capital stock and national total labor force are fully employed by the three sectors. We have

$$K_i(t) + K_e(t) + K_r(t) = K(t), \;\; N_i(t) + N_e(t) + N_r(t) = N(t). \qquad (21)$$

Wealth is owned by households

$$\sum_{j=1}^{J} \bar{k}_j(t)\overline{N}_j = K(t). \qquad (22)$$

We constructed a dynamic general equilibrium model with endogenous wealth, human capital and knowledge. Which is composed of any number of national economies. Markets are perfectly competitive. The model is built on the basis of some main ideas in economic growth theory. Structurally it includes some models as special cases. For instance, if we fix human capital and knowledge and national economies are identical, our model is structurally similar to the neoclassical growth models by Solow (1956), Uzawa (1961, 1963). Our model is similar to the Uzawa-Lucas model if we fix knowledge and assume identical national economies (Uzawa, 1965; Lucas, 1988). If human capital is fixed, it is by the Zhang's model of knowledge growth with research (Zhang, 1992). If human capital and knowledge are fixed, our model is similar to the Oniki-Uzawa model.

3. Economic Dynamics

We first show that in general case the dynamics of the world economy can be expressed by a $2J+1$ dimensional differential equations system. We introduce a new variable $z(t)$

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

Lemma

The dynamics of $J-$type household economy is governed by the following $2J+1$ differential equations with $z(t)$, $Z(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \cdots, H_J(t))$, as the variables

$$\dot{z}(t) = \Omega_z\left(z(t),\ Z(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),$$

$$\dot{Z}(t) = \Omega_Z\left(z(t),\ Z(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),$$

$$\dot{\bar{k}}_j(t) = \Omega_{jk}\left(z(t),\ Z(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),\quad j = 2, \ldots, J,$$

$$\dot{H}_j(t) = \Omega_{jH}\left(z(t),\ Z(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),\quad j = 1, \ldots, J, \tag{23}$$

in which functions $\Omega(t)$ are uniquely determined by variables $z(t)$, $Z(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, as shown in the Appendix. For any given solution $z(t)$, $Z(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, all the other variables are uniquely determined by the following procedure: $r(t)$ by (A2) $\rightarrow$ $w(t)$ by (A3) $\rightarrow$ $p(t)$ by (A4) $\rightarrow$ $\bar{k}_1(t)$ by (A17) $\rightarrow$ $N_i(t)$ by (A15) $\rightarrow$ $N_r(t)$ by (A12) $\rightarrow$ $N(t)$ by (A14) $\rightarrow$ $N_e(t)$ by (A9) $\rightarrow$ $K_m(t)$, $m = i, s, r$, by (A1) $\rightarrow$ $F_e(t)$ by (13) $\rightarrow$ $\bar{y}_j(t)$ by (A8) $\rightarrow$ $c_j(t)$, $s_j(t)$, $T_{je}(t)$ by (11) $\rightarrow$ $T_j(t) = T_0 - T_{je}(t)$ $\rightarrow$ $F_j(t)$ by (2).

The lemma provides a computational procedure to plot movement of the economy with any number of households. The dynamical system contains many nonlinear equations. As we cannot give provide general analytical solutions, we use computer to simulate the system with proper initial conditions. We choose $T_0 = 1$ and $\delta_Z = 0.03$. We specify the other parameters as follows

$$\tau = 0.01,\ \alpha_i = 0.35,\ \alpha_e = 0.31,\ \alpha_{0r} = 0.5,\ \beta_{0r} = 0.5,\ \delta_k = 0.04$$

$$\begin{pmatrix}\bar{N}_1\\ \bar{N}_2\\ \bar{N}_3\end{pmatrix} = \begin{pmatrix}10\\ 5\\ 100\end{pmatrix},\ \begin{pmatrix}A_i\\ A_e\\ v_r\end{pmatrix} = \begin{pmatrix}1\\ 1\\ 0.7\end{pmatrix},\ \begin{pmatrix}m_1\\ m_2\\ m_3\end{pmatrix} = \begin{pmatrix}0.5\\ 0.3\\ 0.1\end{pmatrix},\ \begin{pmatrix}m_i\\ m_e\\ m_r\end{pmatrix} = \begin{pmatrix}0.3\\ 0.2\\ 0.2\end{pmatrix},\ \begin{pmatrix}\xi_{10}\\ \xi_{20}\\ \xi_{30}\end{pmatrix} = \begin{pmatrix}0.1\\ 0.1\\ 0.1\end{pmatrix},\ \begin{pmatrix}\lambda_{10}\\ \lambda_{20}\\ \lambda_{30}\end{pmatrix} = \begin{pmatrix}0.8\\ 0.75\\ 0.65\end{pmatrix},$$

$$\begin{pmatrix}\eta_{10}\\ \eta_{20}\\ \eta_{30}\end{pmatrix} = \begin{pmatrix}0.03\\ 0.025\\ 0.02\end{pmatrix},\ \begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \begin{pmatrix}0.01\\ 0.01\\ 0.01\end{pmatrix},\ \begin{pmatrix}\tau_{1k}\\ \tau_{2k}\\ \tau_{3k}\end{pmatrix} = \begin{pmatrix}0.01\\ 0.01\\ 0.01\end{pmatrix},\ \begin{pmatrix}\tau_{1w}\\ \tau_{2w}\\ \tau_{3w}\end{pmatrix} = \begin{pmatrix}0.02\\ 0.01\\ 0.01\end{pmatrix},\ \begin{pmatrix}\tau_{1c}\\ \tau_{2c}\\ \tau_{3c}\end{pmatrix} = \begin{pmatrix}0.02\\ 0.01\\ 0.01\end{pmatrix},$$

$$\begin{pmatrix}m_{1e}\\ m_{2e}\\ m_{3e}\end{pmatrix} = \begin{pmatrix}0.4\\ 0.4\\ 0.2\end{pmatrix},\ \begin{pmatrix}v_{1e}\\ v_{2e}\\ v_{3e}\end{pmatrix} = \begin{pmatrix}1\\ 0.7\\ 0.5\end{pmatrix},\ \begin{pmatrix}b_{1e}\\ b_{2e}\\ b_{3e}\end{pmatrix} = \begin{pmatrix}0.1\\ 0.1\\ 0.1\end{pmatrix},\ \begin{pmatrix}\pi_{1e}\\ \pi_{2e}\\ \pi_{3e}\end{pmatrix} = \begin{pmatrix}0.4\\ 0.6\\ 0.6\end{pmatrix},\ \begin{pmatrix}\delta_{1h}\\ \delta_{2h}\\ \delta_{3h}\end{pmatrix} = \begin{pmatrix}0.03\\ 0.04\\ 0.05\end{pmatrix}. \tag{26}$$

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Group 1, 2 and 3's populations are respectively 10, 50, and 100. Group 1 has the smallest population and Group 3 has the largest population. Group 1, 2 and 3's human capital utilization efficiencies rank from high to low. The representative household of Group 1 applies human capital mostly effectively. The representative household of group 3 applies lest effectively. We specify the values of the parameters, α_{ji}, in the Cobb-Douglas productions approximately equal to 0.3. the tax rates are fixed lowly. The depreciation rate of physical capital is fixed at 4 percent. The depreciation rates of human capital vary between 3 percent and 5 percent. The returns to scale parameters in human capital accumulation are all positive, which implies that human capital accumulation exhibits decreasing returns to scale. We plot the motion of the system with the following initial conditions:

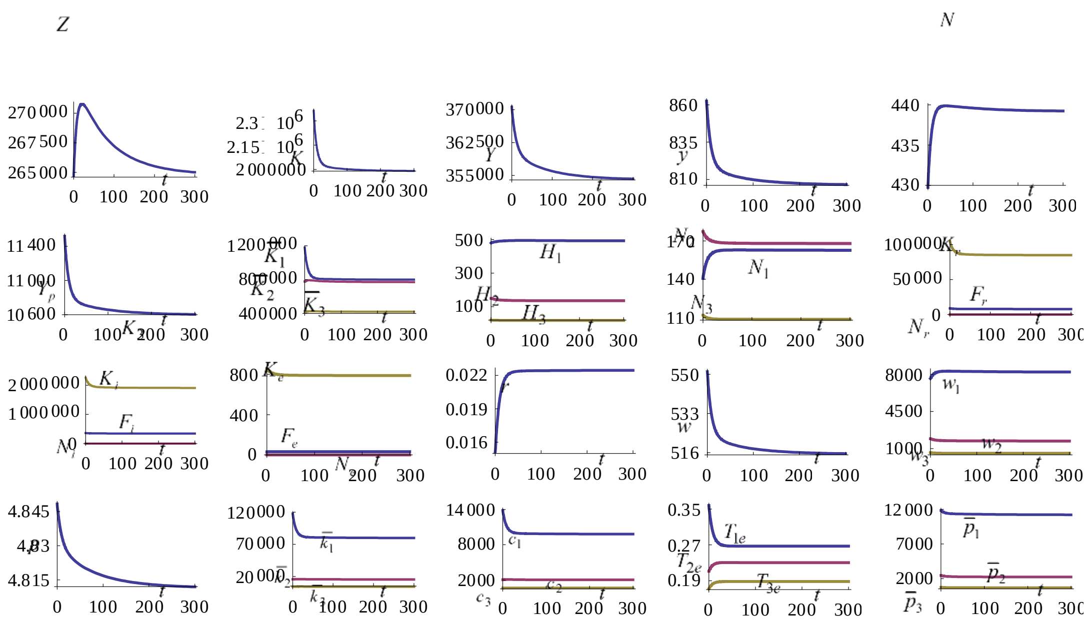
$$z(0) = 0.0001, \ H_1(0) = 480, \ H_2(0) = 150, \ H_3(0) = 20, \ \bar{k}_2(0) = 15500, \ \bar{k}_3(0) = 4100,$$
$$Z(0) = 264700.$$

The system does approach to an equilibrium point in the long term. It starts not far from its long-term equilibrium. Three variables $Y(t)$, $Y(t)$, and $F_r(t)$ are defined as follows

$$Y(t) = F_i(t) + p(t)F_i(t), \ \ y(t) = \frac{Y(t)}{N(t)}, \ \ F_r(t) = v_r Z^{m_r}(t) K_r^{\alpha_{0r}}(t) N_r^{\beta_{0r}}(t).$$

The variable $Y(t)$ measures the total national output, excluding knowledge creation. The variable $y(t)$ is the total output per unit of national labor force. The variable $F_r(t)$ is knowledge output by the research sector. Before the system approaches its equilibrium point, the knowledge stock rises and then falls. The national income falls over time. As the system starts not far from the equilibrium point, most the variables change slightly over time.

Figure 1. The Motion of the Global Economy



It is straightforward to show that the economic dynamic system's equilibrium point is given as follows

$$\begin{pmatrix} Z \\ K \\ N \end{pmatrix} = \begin{pmatrix} 264780 \\ 1.99x10^6 \\ 439.2 \end{pmatrix}, \begin{pmatrix} Y \\ y \\ Y_P \end{pmatrix} = \begin{pmatrix} 343467 \\ 782.1 \\ 10597.6 \end{pmatrix}, \begin{pmatrix} r \\ w \\ p \end{pmatrix} = \begin{pmatrix} 0.023 \\ 515.3 \\ 4.81 \end{pmatrix}, \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 492.5 \\ 135.8 \\ 18.4 \end{pmatrix}, \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 162.6 \\ 167.9 \\ 108.7 \end{pmatrix},$$

$$\begin{pmatrix} \bar{K}_1 \\ \bar{K}_2 \\ \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 799024 \\ 770651 \\ 420469 \end{pmatrix}, \begin{pmatrix} F_i \\ F_e \\ F_r \end{pmatrix} = \begin{pmatrix} 343309 \\ 33 \\ 7943.4 \end{pmatrix}, \begin{pmatrix} N_i \\ N_e \\ N_r \end{pmatrix} = \begin{pmatrix} 428.7 \\ 0.21 \\ 10.28 \end{pmatrix}, \begin{pmatrix} K_i \\ K_e \\ K_r \end{pmatrix} = \begin{pmatrix} 1.91x10^6 \\ 788.1 \\ 84834.9 \end{pmatrix},$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 8380 \\ 1730.8 \\ 559.9 \end{pmatrix}, \begin{pmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \end{pmatrix} = \begin{pmatrix} 11212.3 \\ 2230.9 \\ 687.6 \end{pmatrix}, \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 79902.4 \\ 15413 \\ 4204.7 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 9792 \\ 2034.7 \\ 640.5 \end{pmatrix}, \begin{pmatrix} T_{1e} \\ T_{2e} \\ T_{3e} \end{pmatrix} = \begin{pmatrix} 0.267 \\ 0.23 \\ 0.188 \end{pmatrix}.$$

We also calculate the eigenvalues as follows

-0.136, $-0.125 - 0.122$, -0.08, -0.064, -0.043, -0.012.

We see that the equilibrium is locally stable. This implies that if we start with different initial states not far away from the equilibrium point, the system approaches to the equilibrium point in the long term. This property is important as we can shift parameters to see how the economic system is affected by the exogenous changes in transitory processes and in the long term.

4. Comparative Dynamic Analysis

The previous sector followed the motion of the economic system over time. It is important to ask questions such as how changes in one group's conditions will affect the national economy and income and wealth distributions between groups. This section conducts comparative dynamic analysis. We introduce a variable $\overline{\Delta} x_j(t)$ to represent for the change rate of the variable $x_j(t)$ in percentage due to changes in a parameter value.

4.1. A rise in the research sector's creativity

First, we study how the economic dynamics is changed if the research sector's creativity rises in the following way: $v_r : 0.7 \Rightarrow 0.71$. The simulation results are plotted in Figure 2. The rise of creativity increases the knowledge stock and national wealth. The national output is enhanced. The total labor supply and each group's labor supply are enhanced. The households reduce slightly education time. The price of education is enhanced. The opportunity costs of education for all groups are enhanced. Three groups all augment their human capital and their wage incomes are increased. As the representative household from group 1's wage income and wealth are increased more in percentage than the other two groups, the gaps in income and wealth distributions are enlarged. The production and research sectors expand. The education sector's output changes slightly. The sector's capital input is enhanced and labor input is reduced.

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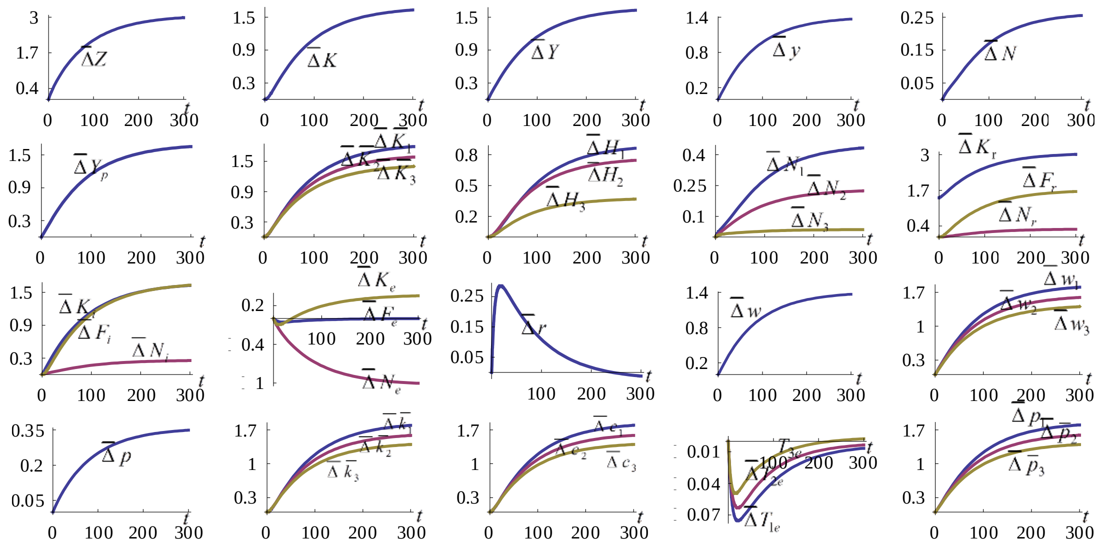
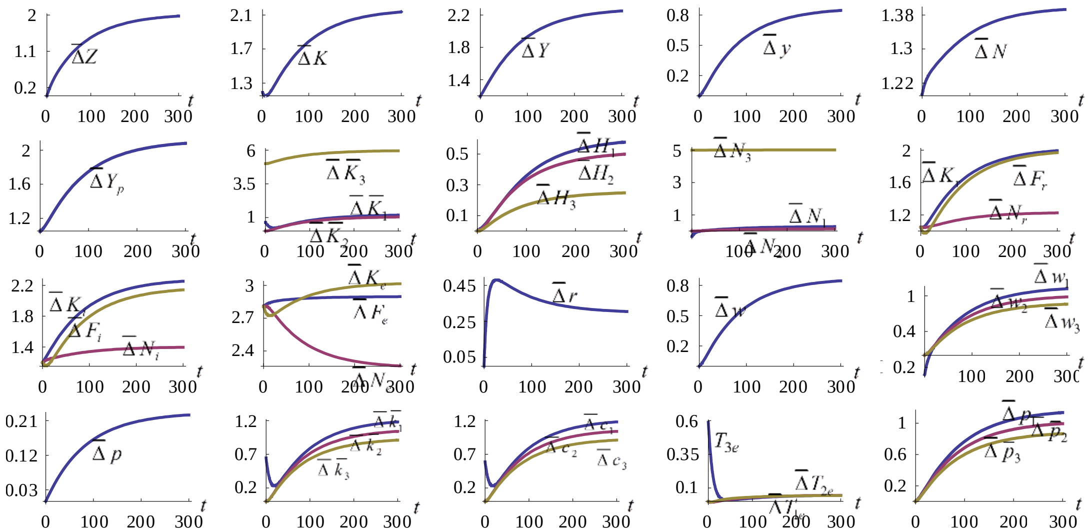
Figure 2. The Research Sector's Creativity Rises



4.2. Group 3's population increases

We now examine how the economic dynamics is changed if group 3's population rises as follows: $\bar{N}_3 : 100 \Rightarrow 105$. The simulation results are plotted in Figure 3. The knowledge stock and national wealth are augmented. The national output is enhanced. The total labor supply and each group's labor supply are enhanced. All the households increase education hours. The price of education is enhanced. The opportunity costs of education for all the groups are enhanced. Three groups all augment their human capital and their wage incomes are increased. The three sectors expand. Each group's per household wealth and consumption levels are enhanced. Our model predicts that a rise in the population will benefit everyone in the knowledge-based economy.

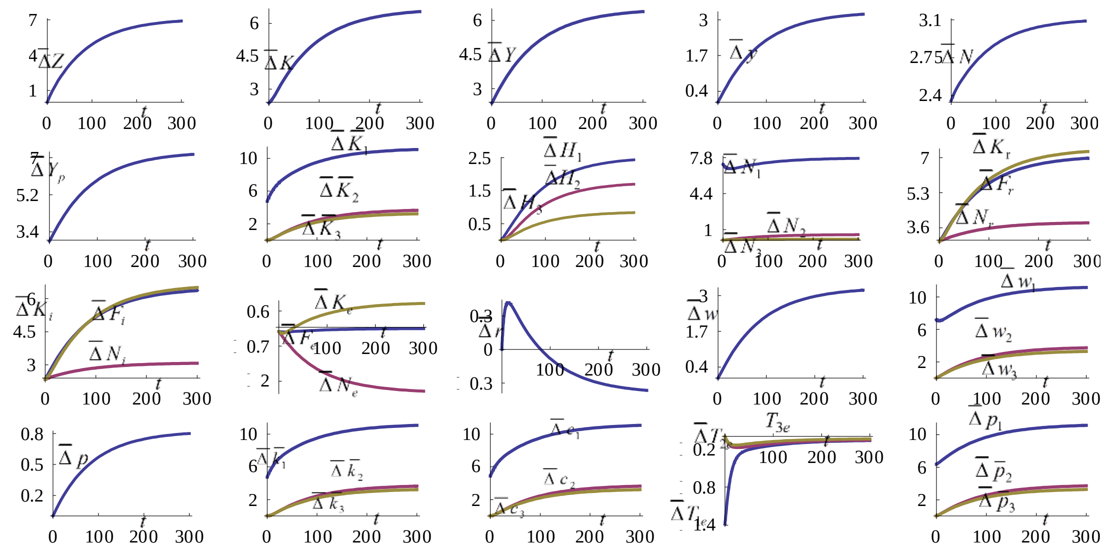
Figure 3. Group 3's Population Increases



4.3. Group 1 applies human capital more effectively

We now study how the economic dynamics is changed if group 1's human capital utilization efficiency is enhanced as follows: $m_1 : 0.5 \Rightarrow 0.51$. The simulation results are plotted in Figure 4. The rise of human capital utilization increases the knowledge stock and national wealth. The national output is enhanced. The total labor supply and each group's labor supply are enhanced. The households reduce education hours in association with rises in the opportunity costs of education for all groups. Three groups all augment their human capital and their wage incomes are increased. The production and research sectors expand. The education sector's output changes slightly. The sector's capital input is enhanced and labor input is reduced.

Figure 4. Group 1 Applies Human Capital More Effectively



4.4. A rise in the tax rate on the production sector

We deal with how the economic dynamics is changed if the government's tax rate on the production sector is increased as follows: $\tau: 0.01 \Rightarrow 0.012$. The simulation results are plotted in Figure 5. The tax income is increased. The research sector is expanded. The knowledge stock and national wealth are augmented. The national output is enhanced. The total labor supply and each group's labor supply are enhanced. All the households initially increase education hours, then reduce education hours, and change little in the long term. The price of education is enhanced. The opportunity costs of education for all the groups are enhanced. Three groups all augment their human capital and their wage incomes are increased. The three sectors expand. Each group's per household wealth and consumption levels are enhanced.

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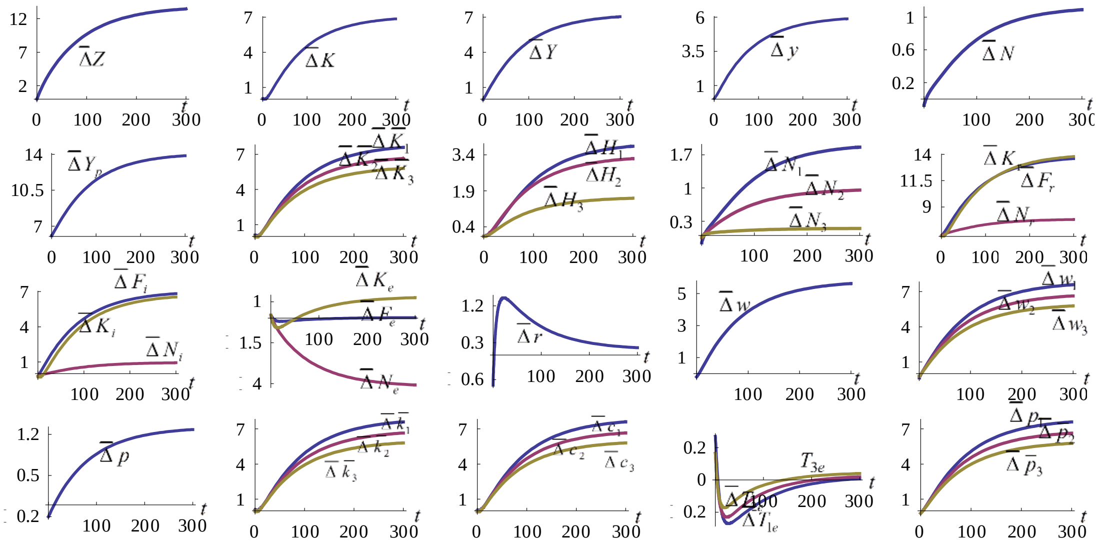
Figure 5. A Rise in the Tax Rate on the Production Sector



4.5. A rise in the tax rate on group 1's consumption of goods

We now examine how the economic dynamics is changed if the government's tax rate on group 1's consumption of goods is increased as follows: $\tau_{1c} : 0.02 \Rightarrow 0.021$. The simulation results are plotted in Figure 6. The tax income is increased. The research sector is expanded. The knowledge stock and national wealth are augmented. The national output is enhanced. The total labor supply and each group's labor supply are enhanced. All the households initially increase education hours, then reduce education hours, and change little in the long term. The price of education is enhanced. The opportunity costs of education for all the groups are enhanced. Three groups all augment their human capital and their wage incomes are increased. The three sectors expand. Each group's per household wealth and consumption levels are enhanced.

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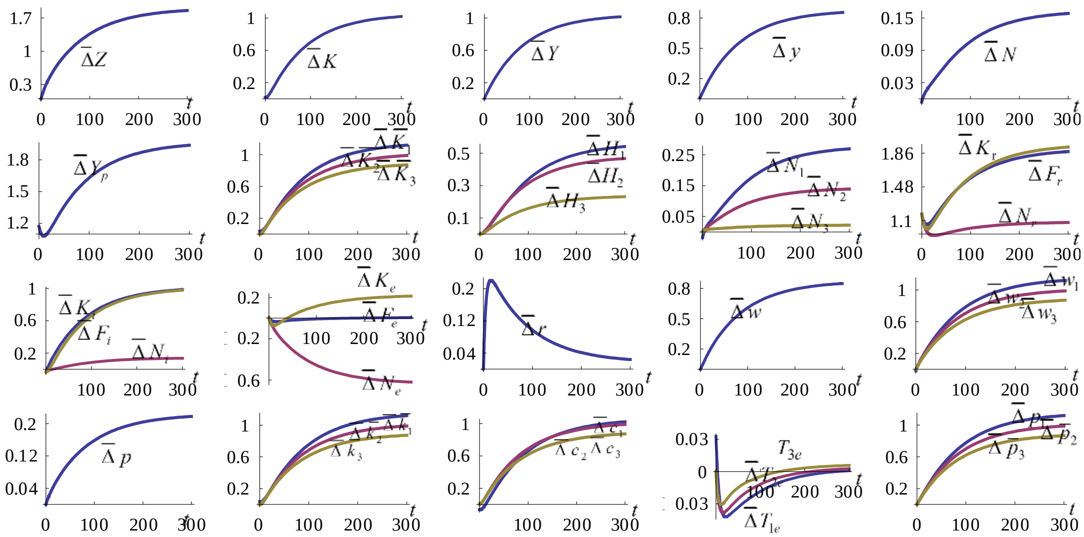

Figure 6. A Rise in the Tax Rate on Group 1's Consumption of Goods

4.6. A rise in group 1's propensity to receive education

We now deal with how the economic dynamics is affected if group 1 enhances the propensity to receive education as follows: $\eta_{01}: 0.03 \Rightarrow 0.031$. The simulation results are plotted in Figure 7. Group 1 spends more time on education. The knowledge stock, national wealth, and national output fall. Group 1's human capital rises initially and falls in the long term. Group 1's labor supply falls and the other two groups' labor supplies fall slightly. The national labor supply falls. The wage incomes are reduced. The price of education falls. The opportunity costs of education for all the groups are reduced. Three groups all reduce their human capital and their wage incomes are decreased. The three sectors shrink. It should be noted that many empirical studies are conducted on relations between education and income. According to Mincer (1974), white males not working on farms have a rise in the earnings by about 7% due to an extra year of education. Psacharopoulos (1994) compares the rates of return to education among 78 countries and finds national differences. According to O'Neill (1995), there is reduction in income dispersion in association with convergence in education levels among the developed economies. Bergh and Fink (2009) don't identify a systematic relation between the structure of higher education and the overall degree income inequality. With the parameter values specified our model predicts that if the rich spend more time on education, the gaps between rich and the poor should be reduced. This occurs as the rich is relatively not very effectively in learning. The national resource is shifted to education because the rich use more resource.

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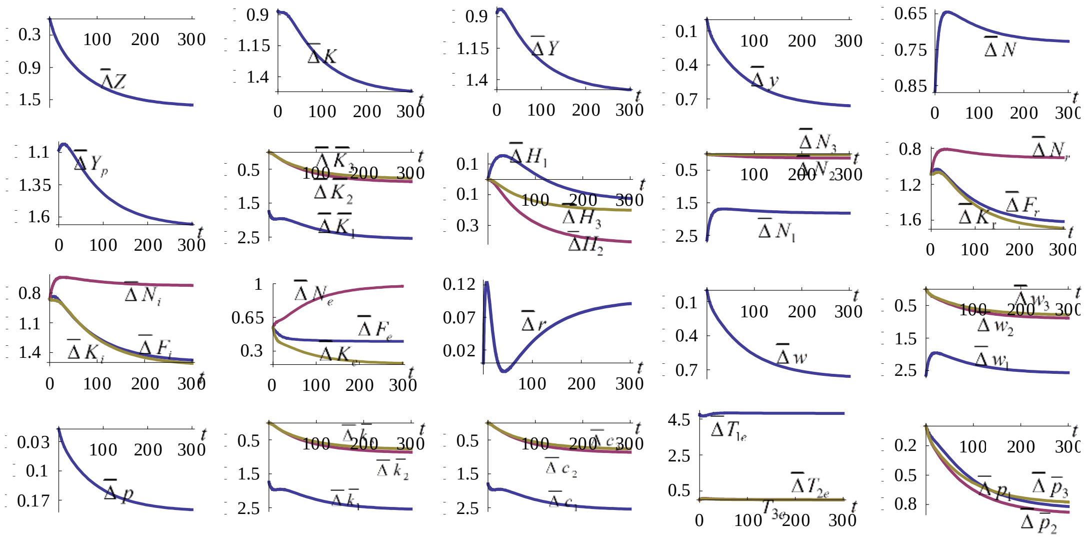

Figure 7. A Rise in Group 1's Propensity to Receive Education

4.7. Group 1 more effectively accumulates human capital with education time

We now deal with how the economic dynamics is affected if group 1 enhances the propensity to receive education as follows: $b_{1e}: 0.1 \Rightarrow 0.11$. The simulation results are plotted in Figure 8. Group 1 spends less time on education. The other two groups' education hours are also slightly reduced. The knowledge stock, national wealth, and national output rise. Group 1's human capital and labor supply rise. The other two groups' human capital and labor supply are slightly augmented. The national labor supply rises. The wage incomes are enhanced. The price of education rises. The opportunity costs of education for all the groups are increased. Three groups all increase their wage incomes. The production and research sectors expand. The education sector's output is slightly affected. The sector employs more capital and less labor force.

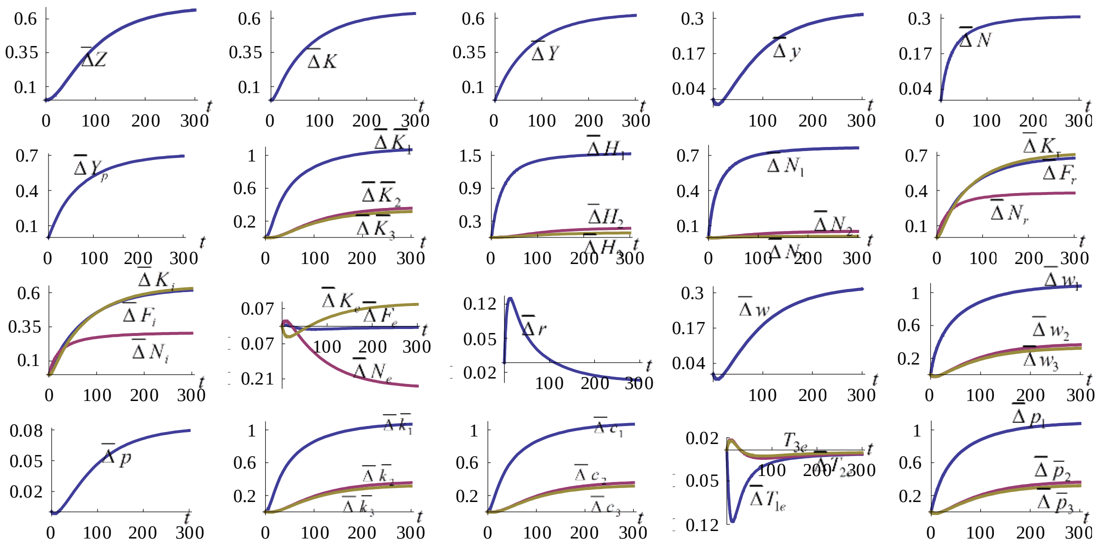

Figure 8. Group 1 More Effectively Accumulates Human Capital with Education Time

5. Conclusions

This study dealt with dynamics of a generalized Walrasian equilibrium theory with endogenous public knowledge, individual human capital, and private wealth. The basic concern is the role of knowledge on human capital and economic growth. The model is a synthesis of a few well-established economic theories with Zhang's approach to modeling human behavior. The temporary general equilibrium economy characterized of fixed knowledge, human capital, and wealth is based on Walrasian general equilibrium theory. Growth mechanism due to capital accumulation is influenced by neoclassical growth theory. Human capital accumulation is a generalization of the Uzawa-Lucas two-sector growth model. Knowledge accumulation is influenced by contemporary endogenous growth theory with endogenous knowledge. The economy is composed of J groups of households and three sectors. The description of economic dynamics is to solve $2J+1$ differential equations. We examined dynamic properties of the model of a 3-group economy with simulation. Comparative dynamic analysis was conducted to reveal transitory and long-term effects of some endogenous changes in policies, preferences, and efficiency of utilizing knowledge or human capital. We showed how changes in the propensity to learn and research policy affect national economic growth and income and wealth distributions. As our model is built on many strict assumptions for simplifying modelling, we can generalize the model in different directions. For instance, we may reasonably have more economic sectors. We may also use more general forms of production functions, utility functions, and/or more realistic functions of creativity and human capital growth.

Appendix: Proving the Lemma

From (3), (14) and (19) we solve

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_m}{\bar{\beta}_m K_m}, \quad m = i, s, e, \qquad \text{(A1)}$$

where

$$\bar{\beta}_m \equiv \frac{\beta_m}{\alpha_m}, \quad m = i, e, r.$$

By (A1) and (4), we get

$$r(Z, z) = \alpha Z^{m_i} z^{\beta_i} - \delta_k \qquad \text{(A2)}$$

where

$$\alpha \equiv \alpha_i \bar{\tau} A_i \bar{\beta}_i^{\beta_i}.$$

From (A1) we have

$$w(Z, z) = \frac{r + \delta_k}{z}. \qquad \text{(A3)}$$

From (13) and (14) we have

$$p(Z, z) = \frac{\bar{\beta}_e^{\alpha_e} w z^{\alpha_e}}{\beta_e A_e Z^{m_e}}. \qquad \text{(A4)}$$

From (2) and (A1), we get

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_e}{\bar{\beta}_e} + \frac{N_r}{\bar{\beta}_r} = z K, \quad N_i + N_e + N_r = N. \qquad \text{(A5)}$$

From (14), we get

$$F_e = \frac{w N_e}{\beta_e p}. \qquad \text{(A6)}$$

Substitute (A6) and (11) into (20)

$$N_e = \frac{\beta_e p}{w} \sum_{j=1}^{J} \bar{N}_j T_{je}. \tag{A7}$$

From (8) we get

$$\bar{y}_j = (1 + \bar{\tau}_{jk} r)\bar{k}_j + \bar{\tau}_{jw} H^{m_j} T_0 w. \tag{A8}$$

Equations (A8) and (A7) imply

$$N_e = n + \sum_{j=1}^{J} n_j \bar{k}_j, \tag{A9}$$

where

$$n_j(Z, z) \equiv \left(\frac{1 + \bar{\tau}_{jk} r}{w}\right)\frac{\eta_j \beta_e p \bar{N}_j}{\bar{p}_j}, \quad n(Z, z) \equiv \beta_e p T_0 \sum_{j=1}^{J} \frac{\eta_j \bar{\tau}_{jw} H_j^{m_j} \bar{N}_j}{\bar{p}_j}.$$

Equations (A8) and (11) imply

$$c_j = (1 + \bar{\tau}_{jk} r)\xi_j \bar{k}_j + \bar{\tau}_{jw} \xi_j H_j^{m_j} T_0 w. \tag{A10}$$

Equations (17) and (19) imply

$$N_r = \frac{\beta_r \tau F_i}{w} + \beta_r \sum_{j=1}^{J}\left(\frac{\tau_{jc} c_j \bar{N}_j}{w} + \frac{\tau_{jk} r \bar{k}_j}{w} \bar{N}_j + \tau_{jw} H_j^{m_j} T_0 \bar{N}_j - \tau_{jw} H_j^{m_j} T_{je} \bar{N}_j\right). \tag{A11}$$

Insert (11) in (A11)

$$N_r = \frac{\beta_r \tau F_i}{w} + \beta_r \sum_{j=1}^{J}\left(\theta_{0j} \bar{y}_j + \frac{\tau_{jk} \bar{N}_j r \bar{k}_j}{w} + \tau_{jw} H_j^{m_j} T_0 \bar{N}_j\right),$$

where

$$\theta_{0j} \equiv \left(\frac{\tau_{jc} \xi_j}{w} - \frac{\eta_j \tau_{jw} H_j^{m_j}}{\bar{p}_j}\right)\bar{N}_j.$$

Substitute (3) and (A10) into the above equation

$$N_r = \frac{\beta_r \tau N_i}{\beta_i \bar{\tau}} + \theta + \sum_{j=1}^{J} \theta_j \bar{k}_j , \tag{A12}$$

where

$$\theta(Z, z) \equiv \beta_r T_0 \sum_{j=1}^{J} H_j^{m_j} \left(\tau_{jw} \bar{N}_j + \bar{\tau}_{jw} \theta_{0j} w\right), \quad \theta_j(Z, z) \equiv \beta_r \left((1 + \bar{\tau}_{jk} r)\theta_{0j} + \frac{\tau_{jk} \bar{N}_j r}{w} \right).$$

Equations (A8) and (11) imply

$$T_j = T_0 - \frac{\eta_j \bar{\tau}_{jw} H^{m_j} T_0 w}{\bar{p}_j} - \frac{\eta_j (1 + \bar{\tau}_{jk} r)\bar{k}_j}{\bar{p}_j}. \tag{A13}$$

Substitute (A13) into (1)

$$N = \tilde{n} - \sum_{j=1}^{J} \tilde{n}_j \bar{k}_j , \tag{A14}$$

where

$$\tilde{n} \equiv \sum_{j=1}^{J} \left(1 - \frac{\eta_j \bar{\tau}_{jw} H^{m_j} w}{\bar{p}_j} \right) H_j^{m_j} \bar{N}_j T_0 , \quad \tilde{n}_j \equiv \frac{(1 + \bar{\tau}_{jk} r)\eta_j H_j^{m_j} \bar{N}_j}{\bar{p}_j}.$$

Substitute (A14), (A12), and (A9) into (A5)

$$N_i = \bar{n} + \sum_{j=1}^{J} \bar{n}_j \bar{k}_j , \tag{A15}$$

where

$$\bar{n} \equiv (\tilde{n} - n - \theta)\left(1 + \frac{\tau \beta_r}{\beta_i \bar{\tau}} \right)^{-1}, \quad \bar{n}_j \equiv -\left(1 + \frac{\tau \beta_r}{\beta_i \bar{\tau}} \right)^{-1} (\tilde{n}_j + \theta_j + n_j).$$

Equations (A5) and (22) imply

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_e}{\bar{\beta}_e} + \frac{N_r}{\bar{\beta}_r} = z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j . \tag{A16}$$

Substitute (A9), (A12), and (A15) into (A17)

$$\bar{k}_1 \equiv \Omega_0\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right) \equiv \left[\tilde{\theta} + \sum_{j=2}^{J}\left(\beta \bar{n}_j + \frac{n_j}{\bar{\beta}_e} + \frac{\theta_j}{\bar{\beta}_r} - z \bar{N}_j\right)\bar{k}_j\right]\left(z \bar{N}_1 - \beta \bar{n}_1 - \frac{n_1}{\bar{\beta}_e} - \frac{\theta_1}{\bar{\beta}_r}\right)^{-1}, \quad \text{(A17)}$$

where

$$\beta \equiv \frac{1}{\bar{\beta}_i} + \frac{\tau \beta_r}{\bar{\beta}_r \beta_i \bar{\tau}}, \quad \tilde{\theta} \equiv \beta \bar{n} + \frac{n}{\bar{\beta}_e} + \frac{\theta}{\bar{\beta}_r}.$$

We determine all the variables as functions of z, Z, $\left(H_j\right)$ and $\left\{\bar{k}_j\right\}$: r by (A2) $\rightarrow$ w by (A3) $\rightarrow$ p by (A4) $\rightarrow$ $\bar{k}_1$ by (A17) $\rightarrow$ N_i by (A15) $\rightarrow$ N_r by (A12) $\rightarrow$ N by (A14) $\rightarrow$ N_e by (A9) $\rightarrow$ K_m, $m = i, s, r$, by (A1) $\rightarrow$ F_e by (13) $\rightarrow$ $\bar{y}_j$ by (A8) $\rightarrow$ c_j, s_j, T_{je} by (11) $\rightarrow$ $T_j = T_0 - T_{je}$ $\rightarrow$ F_j by (2). From the procedure, (12), (15) and (16) we have

$$\dot{\bar{k}}_1 = \Omega_0\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right), \quad \text{(A18)}$$

$$\dot{\bar{k}}_j = \Omega_{jk}\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right), \quad j = 2, \ldots, J,$$

$$\dot{H}_j = \Omega_{jH}\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right), \quad j = 1, \ldots, J,$$

$$\dot{Z} = \Omega_Z\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right). \quad \text{(A19)}$$

Taking derivatives of (A17) with respect to time yields

$$\dot{\bar{k}}_1 = \frac{\partial \bar{k}_1}{\partial z}\dot{z} + \sum_{j=1}^{J}\Omega_{jH}\frac{\partial \bar{k}_1}{\partial H_j} + \sum_{j=2}^{J}\Omega_{jk}\frac{\partial \bar{k}_1}{\partial \bar{k}_j} + \Omega_Z\frac{\partial \bar{k}_1}{\partial Z}, \quad \text{(A20)}$$

where we use (A19). From (A18) and (A20) we solve

$$\dot{z} = \Omega_z\left(z, Z, \left(H_j\right), \left\{\bar{k}_j\right\}\right) \equiv \left(\Omega_0 - \sum_{j=1}^{J}\Omega_{jH}\frac{\partial \bar{k}_1}{\partial H_j} - \sum_{j=2}^{J}\Omega_{jk}\frac{\partial \bar{k}_1}{\partial \bar{k}_j} - \Omega_Z\frac{\partial \bar{k}_1}{\partial Z}\right)\left(\frac{\partial \bar{k}_1}{\partial z}\right)^{-1}. \quad \text{(A21)}$$

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